Calculus AB
Definitions

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Definition 1. Let $a \in \mathbb{R}$.

A *neighborhood* of a is a set which contains an open interval which contains a.

A *deleted neighborhood* of a is a set of the form $U \smallsetminus \{a\}$, where U is a neighborhood of a.

Definition 2. Let f be defined in a deleted neighborhood of $a \in \mathbb{R}$. Let $L \in \mathbb{R}$.

We say that the *limit* of $f(x)$ as x approaches a equals L, and write $\lim_{x \to a} f(x) = L$, if for every neighborhood V of L there exists a deleted neighborhood U of a such that f maps U into V, that is,

$$x \in U \quad \Rightarrow \quad f(x) \in V.$$

Definition 3. Let f be defined in a neighborhood of $a \in \mathbb{R}$.

We say that f is *continuous* at a if

(a) $f(a)$ exists;

(b) $\lim_{x \to a} f(x)$ exists;

(c) $\lim_{x \to a} f(x) = f(a)$.

Definition 4. Let f be defined in a neighborhood of $a \in \mathbb{R}$.

We say that f is *differentiable* at a if

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

exists. If this limit exists, it is called the *derivative* of f at a, and is denoted $f'(a)$.

Definition 5. Let f be defined in a neighborhood D of $a \in \mathbb{R}$.

We say that f has a *local maximum* at a if there exists a deleted neighborhood $U \subset D$ of a such that

$$x \in U \quad \Rightarrow \quad f(x) < f(a).$$

In this case, we call $f(a)$ the *local maximum value.*

We say that f has a *local minimum* at a if there exists a deleted neighborhood $U \subset D$ of a such that

$$x \in U \quad \Rightarrow \quad f(x) < f(a).$$

In this case, we call $f(a)$ the *local minimum value.*

We say that f has a *local extremum* at a if f has either a local minimum or a local maximum at a.

Definition 6. Let f be defined on an interval I.

We say that f is *increasing* on I if, for every $x_1, x_2 \in I$, with $x_1 < x_2$, we have $f(x_1) < f(x_2)$.

We say that f is *decreasing* on I if, for every $x_1, x_2 \in I$ with $x_1 < x_2$, we have $f(x_1) > f(x_2)$.

Definition 7. Let f be differentiable in a deleted neighborhood of $c \in \mathbb{R}$.

We say that c is a *critical point* of f if either $f'(c) = 0$, or if $f'(c)$ does not exist.

Definition 8. Let f be defined on an interval I.

We say that f is *concave up* on I if for every $x_1, x_2 \in I$ with $x_1 < x_2$, the chord between $(x_1, f(x_1))$ and $(x_2, f(x_2))$ lies above the graph of f.

We say that f is *concave down* on I if for every $x_1, x_2 \in I$ with $x_1 < x_2$, the chord between $(x_1, f(x_1))$ and $(x_2, f(x_2))$ lies below the graph of f.

Definition 9. Let f be defined in a neighborhood of $a \in \mathbb{R}$.

We say that f has a *point of inflection* at a if f has a tangent line at a, and the concavity of f changes at a.